HELSINKI SCHOOL OF ECONOMICS (HSE)
Department of Economics

TRANSFERABILITY ANALYSIS OF A MACROECONOMETRIC MULTI-COUNTRY MODEL

Economics
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TIIVISTELMÄ

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MAKROEKONOMETRISEN MONEN MAAN MALLIN SIIRRETTÄVYYSANALYYSI

TUTKIMUKSEN TAVOITTEET

Tutkimuksen tavoitteena oli tutustua makroekonometriseen monen maan malliin sekä tutkia mahdollisuutta valmiin mallin käyttämiseen toisessa viitekehyksessä.

AINEISTO JA MENETELMÄT

John Taylor julkaisi vuonna 1993 rationaalisiin odotuksiin ja porrastettuihin palkkasopimuksiin perustuvan mallin, jota tässä työssä tutkittiin. Se on seitsemän maan malli, mutta sitä pidettiin tarpeeksi yksinkertaisena mallin siirrettävyyden tutkimiseen. Malliin tutustuttiin huolella, jotta siirrettävyyttä voitaisiin sitten tutkia. Testiestimoinnit tehtiin Brasiliasta kerätyllä aineistolla.

TULOKSET

Alustavat testiestimoinnit antoivat lupaavia tuloksia mallin siirrettävyydestä. Estimoinnit tulisi kuitenkin saattaa päätökseen, ennen kuin lopullisia johtopäätöksiä voidaan tehdä. Tämä koskee varsinkin Taylorin palkkamallia, sillä sitä on kritisoitu, ja on esitetty myös vaihtoehtoisia tapoja mallintaa palkkasopimuksia. Mallin siirtäminen on joka tapauksessa vaikea tehtävä, sillä se vaatii paljon asiantuntijaharkintaa, jota on yleensä käytetty jo alkuperäisen mallinkin laatimisessa.

AVAINSANAT

Makrotaloustiede, makroekonometrinen, monen maan malli, rationaaliset odotukset

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ABSTRACT

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TRANSFERABILITY ANALYSIS OF A MACROECONOMETRIC MULTI-COUNTRY MODEL

THE AIM OF THE STUDY

The aim of the study was to become familiarized with a macroeconomic multi-country model and analyze the possibility of using a ready-made model in another context.

MATERIAL AND METHODS

The model studied was published by John Taylor in 1993. It is based on rational expectations hypothesis and staggered wage setting. It is a model with seven countries, but was considered simple enough to test the model transferability. The model was studied thoroughly in order to test the transferability. Data for Brazil was collected and test estimations were conducted.

RESULTS

The preliminary test estimations gave encouraging results about the model transferability. However, the test estimations should be completed before drawing final conclusions. This applies especially to Taylor's wage setting model, since there has been criticism towards it, and alternative specifications have been proposed. In any case, transferring a model is not a simple task but requires expert judgement that has usually been used already in compiling the original model.

KEYWORDS

Macroeconomics, macroeconometric, multi-country model, rational expectations

Contents

Chapter 1

Introduction

1.1 The aim of the study

The aim of this study is to become familiarized with the way of building macroeconometric multi-country models and how to use them by simulation to study behaviour of economic variables. This will be achieved by studying Taylor's (1993) multi-country model in detail and discussing the possibilities of using it in a different context. Also, an exercise in estimating the equation parameters is conducted.

1.2 Earlier research

Macroeconometric multi-country models have been developed since 1968. Since then, several attempts have been made and the research is still going on, with one of the latest models being the International Monetary Fund's Global Economy Model GEM. Rational expectations hypothesis was presented by John Muth in 1961, and it was introduced to multi-country models in the 1980s.

1.3 Conclusions of the study

Macroeconomic modelling is not a simple task. Adding several countries makes it even more demanding. There is a compromise between the simplicity and intelligibility of the model versus its versality.

Macroeconomic multi-country models can be used to study effects of macroeconomic policies, if the frame of reference is well-defined and restricted.

Transferring a macroeconometric model to a different context is not easy and it can be easier to build a new model taking the available data into consideration.

1.4 The structure of the study

Multi-country models and the motivation for their use is described in chapter 2. Also, Taylor's (1993) multi-country model is presented in detail. Chapter 3 describes rational expectations models and the methods to solve them. In chapter 4, the estimation of Taylor's multi-country model is presented. Moreover, the simulation procedure is described, and the main results of Taylor's simulations are presented. In chapter 5, the applicability of Taylor's model is discussed and as a case example, parameters for Brazilian data are estimated. Chapter 6 concludes the study. The appendix includes a list of the model variables and the model equations.

Chapter 2

Macroeconometric multi-country models

2.1 Macroeconometric vs. macroeconomic

In the beginning, a distinction is to be made between the terms *macroeconomic* and *macroeconometric*. They are sometimes used both incorrectly and interchangeably, creating some confusion to the reader, so a short clarification was deemed necessary.

Yap (2002, p. 2) gives a short and good explanation, where he states that the term macroeconomic has sometimes been used to denote both macroeconometric models and computable general equilibrium (CGE) models. However, macroeconometric models need not be economy-wide in nature, so not all macroeconometric models are necessarily macroeconomic.

Furthermore, Whitley (1994, p. 20) gives a good definition to a macroeconometric *model*: "A macroeconometric model is a mathematical representation of the quantitative relationships among macroeconomic variables such as employment, output, prices, government expenditure, taxes, interest rates and exchange rates." He emphasizes its difference from stylised textbook models that do not attempt to quantify the strengths of the relationships.

2.1.1 Macroeconometric models

Macroeconometric models are used to help economists answer macroeconomic policy questions such as effects of monetary policy, fiscal policy or exchange-rate policy. The different policies are conducted by using *policy rules*, which are defined as "systematic responses of the policy instruments to the state of economy" (Taylor 1993), or by using discretion. The policy rules need not necessarily be mechanical formulas, they can also be made under judgment, like a nominal-income rule. However, this is still different form pure discretion, where the settings for the instruments of policy are determined from scratch each period. (Taylor 1993)

Macroeconometric models have been used from 1960s by the private sector, in academic institutions and in government and official agencies to analyse the economy and to evaluate macroeconomic policies, and to make predictions about the likely future behaviour of the economy (Whitley 1994).

There are different models for different uses. Models may have different aims or different theoretical viewpoints. In constructing the model, there always exists the conflict between the desire to explain more, and hence to construct a bigger model, and the wish to make the model more manageable, both in terms of understanding the model and in reducing its maintenance costs. (Whitley 1994)

An important factor in the development of macroeconometric models has been the availability and power of computer hardware, which has increased rapidly, thus lifting many constraints to the modelling (Whitley 1994).

2.1.2 Multi-country models

When a model is constructed of a single country, it cannot be used to evaluate mixtures of fiscal policy and monetary policy or the choice of an exchange-rate policy. That is where models of two or more countries are constructed and the limitations removed. (Taylor 1993)

The first major effort on multi-country modelling was made by the establishment of Project Link in 1968. The aim of the project was to take independent country models and then link them together through merchandise flows and prices. (Whitley 1994)

Whitley's (1994) text has a good review of the history of macroeconometric (multi-country) modelling. Worth mentioning is the Brookings model in the USA in the 1960s. The first version of the model had about 200 equations, to be expanded to 400 later on and the project involved about 30 top economists.

John Taylor's (1993) multi-country model consists of 17 different equations, which are introduced in this text. Using the equations for the G-7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States) yields a total of 98 equations. In this model, staggered wage- and price-setting framework is used, as well as a framework that allows some wages to be set in a synchronized fashion, since one of the G-7 countries, Japan, has most of its wage decisions made in the spring and early summer.

Taylor's multi-country model disaggregates consumption, investment, import and export decisions and explicitly shows how these depend on estimates of future income prospects, expected sales, real interest rates, and exchange rates. Interest rates are determined in a worldwide capital market in which capital flows freely between countries. The central idea to his multi-country model is a theory of the link between aggregate demand and production based on the staggered wage and price setting framework.

Taylor had already constructed an earlier version of the model in 1986 (Bryant et al. 1988). The commentary for that paper approved Taylor's incorporation of rational expectations, which was still a novelty. Taylor had also used the model in his earlier paper (Taylor 1989), where he reports simulation results between fixed and flexible exchange rate regimes.

There are several multi-country models that are used even today, like Fair's (1984, 2004) and Deutsche Bundesbank's (2000). Deutsche Bundesbank's paper has a table of most important multi-country models until the year 2000.

The International Monetary Fund (IMF) has developed multi-country models during recent years. Multimod was first documented in 1988 (Masson et al. 1988), and its most recent version is Mark III (Laxton et al. 1998). Multimod is, like Taylors model, based on rational expectations hypothesis. One of the most recent efforts on the multi-country modelling is IMF's Global Economy Model GEM (Bayuoumi 2004). It has an innovative feature of having a flexible structure, which enables the possibility to include or exclude features at the user's discretion. In Bayoumi's paper a short history of multi-country models is also presented.

2.1.3 The purpose of the study

The purpose of this study is to analyze and discuss Taylor's multi-country model (Taylor 1993) and its equations in some detail and then discuss the model's applicability to another country or countries. The mathematical estimation procedure of the model will not be studied in detail, in order to limit the discussion. However, a basic review of rational expectations will be given as well as the outlines of the estimation procedure.

2.2 Taylor's model

This section introduces Taylor's (1993) model with the same notation that he used in his book. The estimation procedure of parameters is presented in a separate chapter, after introducing rational expectations models and their estimation methods.

Taylor has built the model to suit the data he had available. The following variables are covered with equations: wage setting, aggregate price adjustment, import and export prices, exchange rates and interest rates, term structure of interest rates, consumption demand, fixed and inventory investment, exports and imports, and money demand.

In the following, the variables beginning with a capital "L" are natural logarithms of the actual variables. The subscript i is a country-specific subscript. A negative number in parenthesis after a variable represents a corresponding lag, a positive number indicates the expected value of the variable over the respective number of periods in the future.

2.2.1 Wage setting

The wage setting equations are based on Taylor's (1980) staggered wage-setting model. In the actual multi-country model the wage setting is synchronized. The equations are based on two key assumptions: 1. wage contracts are staggered which means that not all wage decisions are made at the same time, and, 2. when making wage decisions, firms (and unions) look at the wage rates which are set in other firms and which will be in effect during their contract period. Because of the staggering, some firms will have established their wage rates before current negotiations and others will establish theirs in future periods. This means that the firms have to look both forward and backward in time. (Taylor 1980)

In an earlier paper, Taylor (1979b) finds out that relatively short contracts are capable of displaying empirically observed serial persistence. Although other models might explain these correlations just as well, this type of model with relatively short contracts appears to be consistent with the data.

The equations are of the following form:

$$\begin{aligned} LX_i &= \pi_{i0}LW_i + \pi_{i1}LW_i(+1) + \pi_{i2}LW_i(+2) + \pi_{i3}LW_i(+3) \\ &\quad + \alpha_i[\pi_{i0}YG_i + \pi_{i1}YG_i(+1) + \pi_{i2}YG_i(+2) + \pi_{i3}YG_i(+3)], \end{aligned} \tag{2.1}$$

where LX is the log of the contract wage, LW is the log of the average wage, and Y is the output gap. Moreover,

$$\begin{aligned} LW_i &= \pi_{i0}LX_i + \pi_{i1}LX_i(-1) + \pi_{i2}LX_i(-2) + \pi_{i3}LX_i(-3) \\ &\text{and} \\ YG &= \beta_1 YG(-1) + \beta_2 YG(-2) + \beta_3. \end{aligned}$$

Taylor (1980, p. 18) argues that the expectations component of the wage setting model depends on the aggregate-demand policy rule. Due to the dependence, the policy implications of the model are much different from the models that either do not have expectations component, that is, wage determination is purely backward looking, or whose expectations component is based on adaptive or extrapolative expectations schemes. He also adds that price stabilization appears to be very costly when expectations are not rational or contracts do not look forward. The difference between models with and without rational expectations indicates that rational expectations matter greatly for macroeconomic stabilization (Taylor 1980, p. 20).

2.2.2 Aggregate price adjustment

Markup pricing is within the price equation, where prices are assumed to be set as a markup over wages and other costs. Import prices affect the costs of inputs to production, and the markup over wage costs. The depreciations of the currency have direct inflationary consequences in the depreciating country, and deflationary effects abroad through the latter effect.

The price behaviour equation is the following:

$$\begin{aligned} LP_i &= h_{i0} + h_{i1}LP(-1) + h_{i2}LW_i + h_{i3}LPIM_i(-1) + h_{i5}T + U_{pi} \\ U_{pi} &= \rho_{pi}U_{pi}(-1) + V_{pi} \\ &\quad \text{with } h_{i1} + h_{i2} + h_{i3} = 1, \end{aligned} \tag{2.2}$$

where LP is the log of aggregate price, LW is the log of the aggregate wage, $LPIM$ is the log of the import-price index, and T is a time trend. The lagged dependent variable was entered to capture slow adjustment of output prices to changes in costs. Taylor mentiones that output gap was also used as a variable, but its effect was found to be insignificant and it was dropped out of the equation. Discerning reader may notice that parameter h_{i4} is missing from the equation, it was probably output gap's parameter. Homogeneity conditions were imposed on the equation by forcing the parameters h_1–h_3 to sum up as one. This means that one-percent increases in both wages and import prices leads to a one-percent increase in output prices. The error term is a first-order autoregressive process.

2.2.3 Import and export prices

Imports into a country depend partially on the price of imports relative to the price of domestically produced goods, and exports depend similarly on the price of those exports compared with the prices of competitive goods produced abroad.

Import prices

Import prices are assumed to be related to an average of foreign prices in domestic currency. In a multi-country model, the price of imports is a weighted average of foreign prices. Import prices adjust with a long lag to changes in foreign prices, especially when the change is due to exchange-rate movements. This is taken into account by adding the lagged dependent variable into the equation. Domestic prices may also have an effect on import prices, but in Taylor's research, the effect was small and statistically insignificant and subsequently omitted from the final equations for simplicity.

The import-price equation has thus the following log-linear form:

$$\begin{aligned} LPIM_i &= k_{i0} + k_{i1} LPIM(-1) + k_{i2} LFP_i + U_{mi} \\ U_{mi} &= \rho_{mi} U_{mi}(-1) + V_{mi} \\ & \quad \text{with } k_{i1} + ki2 = 1, \end{aligned} \tag{2.3}$$

where $LPIM$ is the log of the import price and LFP is the log of the foreign price. The long-run elasticity is constrained to be one, and the error term is again a first-order autoregressive process.

Export prices

Export prices are assumed to be related to the average price of goods produced in importing countries. In this case both domestic and foreign prices (prices in the country where the goods are sold) have an effect on the export prices. Thus,

$$\begin{aligned} LPEX_i &= \beta_{i0} + \beta_{i1} LPIM(-1) + \beta_{i2} LP_i + \beta_{i3} LFP_i + \beta_{i4} T + U_{xi} \\ U_{xi} &= \rho_{xi} U_{xi}(-1) + V_{xi} \\ & \quad \text{with } \beta_{i1} + \beta i2 + \beta_{i3} = 1, \end{aligned} \tag{2.4}$$

where $LPEX$ is the log of the export price, LP is the log of the domestic price index, and LFP is the log of the foreign price index.

2.2.4 Exchange rates and interest rates

Uncovered interest rate parity states that the difference between interest rates in two countries is equal to the expected change in the exchange rate between them over the near future. Time-varying risk premia and other factors can shift this relation. Such relations, along with possible shifts, are shown in

$$\begin{aligned} LE_i &= LE_i(+1) + 0.25 * (RS_i - RS_0) + U_{ei} \\ U_{ei} &= \rho_e U_{ei}(-1) + V_{ei}, \end{aligned} \tag{2.5}$$

where LE is the log of the exchange rate between the two countries (country i and the United States in Taylor's text) and $RS_i - RS_0$ is the short-term interest rate differential between the countries (RS_0 being the United States). All the cross-exchange rates can be derived from the pairwise equations of a country i and a fixed country. The equation has the coefficient of 0.25 because interest rates are measured at annual rates, and the expected change in the exchange rate is over one quarter. Hence, in this equation the coefficients are not estimated. Instead, the residuals are computed to be used in the policy analysis.

These equations imply financial capital mobility. During the time period Taylor used to estimate the equations (1971–1986), the assumption seemed to be valid for most of the countries.

2.2.5 Term structure of interest rates

In this model, Shiller's (1979) linear approximation of the term structure was used. As Shiller states, "the long-term interest rate can be approximately represented as a long average of rationally expected future short-term rates plus a liquidity premium term" (Shiller 1979, p. 1190). The equation (Shiller 1979, p. 1194) has the following form:

$$RL_i = b_{i0} + \frac{1 - b_i}{1 - b_i^9} \sum_{s=0}^{8} b_i^s RS_i(+s), \tag{2.6}$$

where RL is the long-term interest rate, and values of RS represent expected future short-term interest rates.

2.2.6 Consumption demand

The consumption equations are based on a rational expectations forward-looking model, which is discussed in Hall and Taylor (1997). The forward-looking behaviour was modelled empirically by constructing a measure of permanent income, which depends on rational expectations of actual future income. In an earlier paper, Hall (1978) found out that real disposable income has no predictive power for consumption, but Taylor (1993, pp. 87–88) states that since permanent income variable includes both current income and expectations of future income, this does not contradict Hall's result. The equations also include the real interest rate, which depends on the expected rate of inflation. The consumption is disaggregated into durables, non-durables and services, whenever appropriate data is available. The consumption of durables is more volatile than the consumption of services, and also more sensitive to interest rates. Non-durables lie in between on these volatility and sensitivity issues. The general form of the equation is

$$CX_i = c_{i0} + c_{i1}CX_i(-1) + c_{i2}YP_i + c_{i3}RRL_i, \tag{2.7}$$

where CX is either CD for consumer durables, CN for consumer non-durables, CS for consumer services, or C for total consumption, and where YP is permanent income and RRL is the real interest rate. The consumption equation is linear in the levels of the variables. The permanent income variable is defined as

$$YP_i = \sum_{s=0}^{8}(0.9)^s Y_i(+s).$$

Real output is assumed to be the measure of income. The real interest rate RRL is scaled so that its absolute effect grows with the estimated trend in the real economy to prevent the real interest-rate elasticity from declining as consumption grows. Hence, the real interest rate is the difference between the long-term interest rate and the expected rate of inflation multiplied by the exponentially growing trend, which grows at the same rate as potential output,

$$RRL_i = (RL_i - LP_i(4) - LP_i)\exp(gT),$$

where g is the growth rate of potential output.

2.2.7 Fixed investment

Investment demand is assumed to depend on the cost of capital, which is measured by the real interest rate, and on expected future sales, which is assumed to have the same form as the measure of expected future income in the consumption equations. Like the consumption, fixed investment is disaggregated into non-residential equipment, non-residential structures and residential investment, whenever data is available. The general form of the fixed investment equation is

$$IX_i = d_{i0} + d_{i1}IX_i(-1) + d_{i2}YP_i + d_{i3}RRL_i, \tag{2.8}$$

where IX is non-residential equipment INE, non-residential structures INS, non-residential investment IN, residential investment IR, or the total fixed investment IF. The variables YP and RRL are defined as for consumption. Like the consumption equation, the fixed investment equation is linear in the levels of the variables. Lagged investment represents either the cost of adjusting capital or the periods of time to build capital.

2.2.8 Inventory investment

Inventory investment is assumed to have a different functional form than fixed investment. Current sales are assumed to have an effect on the desired level of inventories, and therefore the change in inventories depends on the change in sales. This is called the accelerator model (Samuelson 1939). Taylor also thought that the real interest rate has an effect on inventory investment, and therefore it is also added to the model. The equation for inventory investment is

$$II_i = e_{i0} + e_{i1}II_i(-1) + e_{i2}Y_i + e_{i3}Y_i(-1) + e_{i4}RRL_i, \tag{2.9}$$

where II is inventory investment, Y is real output and RRL is the real interest rate. The lagged dependent variable was also included to reflect adjustment costs. If $e_{i2} > 0$ and $e_{i2} = e_{i3}$, the model is a pure accelerator model and only the *change* in real output affects inventory investment.

2.2.9 Exports and imports

Each country's exports and imports are measured in real terms in the local currency. The measures are the same used to compute a country's GNP of GDP by the expenditure approach in the national income accounts. This means that the trade flows include both merchandise trade and services. In this model, bilateral trade flows between the individual countries participating in the model were not modeled.

$$\begin{aligned} LEX_i &= f_{i0} + f_{i1}LEX(-1) + f_{i2}(LPEX_i - LPIM_i) + f_{i3}LYW_i \quad (2.10) \\ LIM_i &= g_{i0} + g_{i1}LIM(-1) + g_{i2}(LPIM_i - LP_i) + g_{i3}LY_i, \quad (2.11) \end{aligned}$$

where LEX is the log of exports, $LPEX$ is the price deflator for exports, $LPIM$ is the price deflator for imports, LYW is the log of a weighted average of output in the other countries of the model, LIM is the log of imports, LP is the price deflator for output, and LY is the log of real output. Taylor had tried also alternative relative price ratios, but the chosen measures gave most plausible and best fitting equations on the average in all the countries (Taylor 1993, p. 92).

2.2.10 Money demand

The money demand was assumed to have Cagan's (1956) semi-log form, where the log of real money demand is assumed to depend on the log of real income, the level of the short-term interest rate, and the log of lagged real money balances. The equation for money demand is

$$L\left(\frac{M_i}{P_i}\right) = a_{i0} + a_{i1}L\left(\frac{M_i(-1)}{P_i(-1)}\right) + a_{i2}RS_i + a_{i3}LY_i, \quad (2.12)$$

where M is the money supply, P the domestic price index, RS the short-term interest rate, and LY again the log of real output. Lagged real money balances are included to account for slow adjustment.

2.2.11 The whole multi-country model

In order to put the model together, some identities are needed. The income-expenditure identity for aggregate demand Y is the following:

$$Y = C + IF + II + G + EX + IM, \tag{2.13}$$

where consumption C and fixed investment IF are disaggregated according to the data available. Other aggregate variables in the model are weighted output LYW and weighted price LFP. The weighted output LYW is a weighted geometric average of the outputs of the other countries. The weighted price LFP is the weighted geometric average of the prices in the other countries measured by domestic currency that transforms in the logarithmic form into the subtraction of weighted price of the other countries in foreign currency units LPW by weighted exchange rate LEW. The potential output needed to calculate the output gap YP is assumed to be growing exponentially, and the exponential trend is assumed to be constant. The growth rate is estimated by regressing the log of real output on a linear trend. Taylor says that ignoring the study of the policy effects on the growth rate does not compromise the study, since his main focus is on the fluctuations around the trend.

Since the model consists of both linear and nonlinear equations, it cannot be reduced to either log-linear or linear form. Thus, it will have to be estimated numerically. In the next chapter, the estimation procedure for nonlinear rational expectations models is presented. In the following chapter, the estimation of the model is presented.

Chapter 3

Rational expectations models

3.1 Rational expectations

In the previous chapter, the term *rational expectations* was mentioned several times, so an introduction to rational expectations models was deemed appropriate. In this chapter, a solution method called *the extended path method* that is especially appropriate for rational expectations models is also introduced.

Walters (1971) prefers the name "consistent expectations" over "rational expectations", since the expectations are consistent with the relevant economic theory but he argues that rationality is completely another matter. There have been discussions about consumers' rationality and many times it has been shown that a consumer does not necessarily make her decisions in a rational way.

Also, Whitley (1994) mentions that rational expectations would be termed more correctly as "model-consistent expectations", but continues to use the original, more established term.

Griffiths et al. (1993, p. 99) assert that the behavioural assumption of rational expectations is important to modern economic theory. This assumption is that at any time, with all available present information, an individual's expectations about macroeconomic random variables are 'unbiased', that is, correct on average. For example, if conditions leading to inflation are observed, the inflation is anticipated and individuals take measures to protect themselves against if. The rational expectations hypothesis asserts that individuals correctly forecast, on the average, the rate of inflation given the information they have.

Already John Muth (1961) stated in his rational expectations hypothesis paper that rational expectations are simply predictions from economic theory, using the information available at the time the predictions are made. For example, (Holden et at., 1985):,

$$E_t P_{t+1} = P_{t+1} + \epsilon_{t+1},$$

where ϵ should have a mean of zero and be serially uncorrelated. It is important to realise that ϵ is not always zero; expectations can thus be wrong but not systematically under the rational expectations hypothesis. Once an error is observed it does not affect the future expectations since as it is known to be random it contains no new information. This contrasts with the *adaptive expectations* model where past errors modify current expectations.

3.2 Solving rational expectations models

3.2.1 Linear models

The simplest rational expectations model is a linear model with one variable, one expectation and one stochastic shock. The model is the following:

$$y_t = \alpha E_t y_{t+1} + \delta u_t, \tag{3.1}$$

where y_t is the variable, α and δ are the parameters, and E_t is the conditional expectation based on all information until period t, including knowledge of the model. The variable u_t is the shock to the equation. It is assumed to follow the general linear process:

$$u_t = \sum_{t=0}^{\infty} \theta_i \epsilon_{t-i}, \tag{3.2}$$

where θ_i, $i = 0, 1, 2, \ldots$ is a sequence of parameters, and ϵ_t is a serially uncorrelated random variable with zero mean.

Taylor (1986) and Taylor (1993) go thoroughly through the solution of this simple model. In both texts, the solution method is generalized to linear models with several endogenous variables. The benefit of linear models is the possibility to solve the model analytically. Also, the cross-equation constraints can be illustrated algebraically. With nonlinear models, they cannot be any more represented explicitly.

3.2.2 Nonlinear models

In practice, many rational expectations models are not linear. Nonlinear models cannot be usually solved analytically, so one has to use numerical computational methods. Although these methods are computationally different from the methods used to solve linear models, they are conceptually very similar.

3.2.3 Extended path method

A method proposed by Fair and Taylor (1983) has become the most common method of solving large nonlinear rational expectations models. It is an iterative method and it is called the extended path method, because it iterates on future paths of the expected endogenous variables.

A general nonlinear rational expectations model can be written as

$$f_i(y_t, y_{t-1}, \ldots, y_{t-p}, E_{t-1}y_t, E_{t-1}y_{t+1}, \ldots, E_{t-1}y_{t+h}, x_t, \alpha_i) = u_{it} \quad (i = 1, \ldots, n),$$

where y_t is an n-dimensional vector of endogenous variables at time t, x_t is a vector of exogenous variables at time t, E_{t-1} is the conditional expectations operator based on the model and on information through period $t-1$, α_i is a vector of parameters, and u_{it} is a stationary scalar random variable which has mean zero and which may be correlated across equations and over time. Nonlinearity means that the function f_i may be nonlinear in the variables, parameters, and expectations.

Extended path method can be used for solving the model for the vector y_t in terms of its past values and the values of the exogenous variables x_t, and also for obtaining the maximum likelihood estimates of the parameters α_i and the covariance structure of the u_{it} given a series of observations on y_t and x_t. In the following, the basic method for solving the models is represented as in Fair and Taylor (1983). In a later paper, Fair and Taylor (1990) correct some errors of the original paper and also develop their method further.

Let the initial set of values for the expected endogenous variables $E_{s-1}y_{s+r}$ be represented as g_r, $r = 0, 1, \ldots$. In principle, the general model will not have a natural termination date, so an infinite number of these values would be needed to be specified. In practice, only a finite number of these will be used in obtaining a solution with a given finite tolerance range.

The solution method consists of the following five steps:

1. Choose an integer k, which is an initial guess at the number of periods beyond the horizon h for which expectations need to be computed in order to obtain a solution within a prescribed tolerance level δ. Set $E_{s-1}y_{s+r}$ equal to g_r, $r = 0, 1, \ldots, k + 2h$. These initial values will be called $e_r(i, k), i > 0, r = 0, 1, \ldots, k+2h$ for the purpose of describing the iterations.

2. Obtain a new set of values for $E_{s-1}y_{s+r}$, $r = 0, 1, \ldots, k + h$, by solving the model dynamically for y_{s+r}, $r = 0, 1, \ldots, k + h$. This is achieved by setting the disturbances to their expected values (usually zero), using the values $E_{s-1}x_s, \ldots, E_{s-1}x_{s+k+h}$ in place of the actual x's, and using the values $e_r(i, k)$ in place of $E_{s-1}y_{s+r}$. These new guesses are called $e_r(i + 1, k)$, $r = 0, 1, \ldots, k + h$. The solution for each period requires a series of Gauss-Seidel iterations, since the model is nonlinear. These are called *Type I* iterations.

3. Compute for each expectation variable and each period the absolute value of the difference between the new guess and the previous guess, that is, $|e_r(i + 1, k) - e_r(i, k)|$, $r = 0, 1, \ldots, k + h$. If any of these differences are not less than a prescribed tolerance level δ' $(\delta' < \delta)$, that is, if convergence has not been achieved, increase i by 1 and return to step 2. If convergence has been achieved, goto to step 4. Steps 2 and 3 together are a *Type II* iteration step.

   Let $e_r(k)$, $r = 0, 1, \ldots, k + h$ be the vector of the convergent values of a series of Type II iterations.

4. Repeat steps 1 through 3, replacing k by $k + 1$. Compute the absolute value of the difference between each element of $e_r(k + 1)$ and $e_r(k)$, $r = 0, 1, \ldots, h$. If any of these differences are not less than δ, increase k by 1 and repeat steps 1 through 4 (*Type III* iteration step).

Let e_r, $r = 0, 1, \ldots, h$ be the vector of the convergent values of a series of Type III iterations.

5. Use e_r for $E_{s-1}y_{s+r}$, $r = 0, 1, \ldots, h$, and the actual values for x_t to solve the model for period s. This gives the desired solution $\hat{y}_s$.

The above method is used in Taylor (1993) in both obtaining maximum likelihood estimators for the parameters, and in simulation exercises.

Chapter 4

Model estimation and simulations

4.1 Estimation procedure

Taylor's (1993) book reveals that he had to use a lot of discretion in order to get all parameters for all countries. Of course Taylor is an economist with enough experience to be able to decide what actions to take when the estimations do not proceed as expected, or when there appear some anomalities in the estimations, but for a graduate student this kind of discretion is not applicable.

Taylor estimated the equations separately using limited-information maximum likelihood methods. Simultaneous estimation using full-information maximum likelihood methods has been studied (Ripatti 1998) and no significant distinction between the methods was found.

All the parameter values can be found in Taylor's text (1993). Here, the methods of estimation, as well as the difficulties and peculiarities are presented. The estimation is presented in the accuracy deemed sufficient to present the problems encountered during estimation. One purpose of this chapter is also to point out that actual implementation of theory is not as straightforward as one might think or like to believe.

This work does not try to go into details in the econometric methods used in the simulation. Most advanced econometric textbooks cover the methods presented here. See, for example, Maddala (1977). The more advanced methods presented in separate papers are also cited.

4.1.1 Wage setting

The wage setting equations were estimated differently from the other equations, which were estimated with the econometric software TSP, by constructing a separate computer program specifically for the equations. First, the model was reduced and factored into a polynomial product. The estimation itself was conducted with maximum likelihood (ML) method. Taylor had also tried instrumental variable approach where two-stage least squares (2SLS) or Hansen's (1982) generalized method of moments (GMM) estimator is applied and the expectations are replaced by the actual values. These methods, however, gave "wrong" values for the sensitivity parameter in a sense that the sign of the parameter was wrong. Taylor believes that this was due to replacing the expected values with the actual ones, when important timing of expectations is ignored.

4.1.2 Aggregate price adjustment

Like the equation 2.2 indicates, the positive serial correlation was corrected with a first-order autoregressive process for all countries except Germany (see Taylor 1993, pp. 79–80). There were also some minor changes, like using the variable LFP instead of $LPIM$ for Germany and Canada, and adjusting the time trend computation for Canada, Italy and Japan.

The estimation was, however, straightforward compared to, for example, the wage setting equations.

4.1.3 Import and export prices

Similarly to the aggregate price equations, both equations 2.3 and 2.4 have serially correlated shocks. Again, some manual adjustments were made, since the coefficient of the lagged dependent variable was estimated to be greater than one for Canada in the import price equation. Since this would have caused instability in the overall model, Canada's coefficient was set equal to the United States' one. Also, for the export prices, the parameter for variable LFP was not statistically significant for the United States, Canada, or France.

Besides these adjustments, the estimation was similar to the aggregate price adjustment.

4.1.4 Exchange rates and interest rates

The equation 2.5 did not have coefficients to be estimated, but residuals were computed to be used in policy analysis. Actually, the autoregressive coefficient ρ should be estimated, but Taylor decided to use the value of 0.5 for all countries as a "rough average" (Taylor 1993, p. 130).

4.1.5 Term structure of interest rates

Taylor reports using two-stage least squares (2SLS) method for estimation with actual values replacing the expected future values, without stating any particular reason for this. The estimation procedure is argued to be consistent, with the standard error of the parameter being inconsistent due to the serial correlation of the forecast errors in projecting interest rates (Taylor 1993, p. 84). The estimation was successful for all countries except Italy, where the equation parameter was negative but also statistically insignificant, so it was set to zero in the simulations.

4.1.6 Consumption demand

Disaggregated data were available for all countries, except Germany and Italy. For them, total consumption equations were estimated. For the United States, Canada, France, Japan, and the United Kingdom, consumption was disaggregated into durables, non-durables, and services.

The estimation method was generalized method of moments (GMM) for all equations. The real interest rate variable is not significant in all equations, see Taylor (1993, pp. 86–88) for more details. The permanent income variable was significant in all estimated equations.

4.1.7 Fixed investment

Most disaggregated data was available for the United States, where fixed investment was broken into non-residential equipment, non-residential structures and residential structures. France, Japan, and the United Kingdom had data for non-residential and residential investment, and Canada, Germany, and Italy had data for total fixed investment only.

The estimation method was once again GMM. The real interest rate had a negative coefficient in all except one equation, French non-residential investment. Because of this, Taylor decided to omit the variable from that equation altogether. The real interest rate variable is not significant in all equations this time, either, though more so than in the consumption equations. The expected future sales modelled by permanent income was omitted from some equations. (Taylor 1993, pp. 89–91)

4.1.8 Inventory investment

The estimation method was again GMM, except for Japan, where two-stage least squares method (2SLS) was used, although Taylor does not mention why. The results suggest an accelerator model for all countries, except Japan, since the real output coefficients are always with opposite signs and with approximately same absolute value. For Japan, the signs are reversed, and Taylor figures this to be due to a buffer stock role of inventories. The real interest rate is always negative, although not always significant.

4.1.9 Exports and imports

Because the equations were log-linear, the estimation method was Ordinary Least Squares (OLSQ). The sign of the price variable is negative in all equations except for Germany's import demand equation, where Taylor decided to omit the term. Taylor also does not mention that the term is not significant in all equations. On the other hand, the lagged dependent variable is significant in all equations (except one). (Taylor 1993, pp. 93–94)

4.1.10 Money demand

The equations were estimated by 2SLS. Taylor added a time trend for the United States and the United Kingdom starting at the first quarter of 1982 to capture the effects of regulatory change and financial innovation. There appears to be serial significant correlation only with Italian data with Durbin-Watson statistic 1.2 (see Taylor 1993, p. 95).

4.1.11 Conclusions

For some reason, Taylor has used slightly different sample periods in some cases, although he does not justify this by any way. This may be another adjustment he needed in order to carry out all the estimations successfully.

In any case, it is obvious that a considerable amount of expert judgement was needed in order to successfully conduct the entire estimation process. Taylor does not justify every time his decisions, and also does not omit statistically insignificant parameters.

4.2 The simulation exercises

4.2.1 The extended path algorithm

The essence of the simulation procedure is the extended path algorithm (Fair-Taylor), which is presented in the previous chapter. The simulation subroutine solves the model running through three nested loops, with the outer loop being the Type III iteration and the inner loop being the Type I iteration.

The Type I iteration loop solves the model by using Gauss-Seidel iteration technique and applying the initial (baseline) values to the model equations and using rational "guesses" of the future values needed in several equations. The model is solved when the difference of all the variable values between two iteration steps is smaller than the convergence criterion.

After solving the model, the Type II iteration step checks if the expected future values are equal with the values the solved model. If not, the expected values are updated to the values that the Type I iteration finished with and a new Type I iteration loop is run. This is done until convergence.

The final step is to run the Type III iteration step, which consists of increasing the future horizon with one and running all over. This should be only a verification, since the result is not expected to change in the program unlike in the original algorithm, where the horizon would be increased again with one. In the program, unsuccessful Type III step results in program termination.

4.2.2 Simulation preparations

The first task after finishing the model estimation was to try to understand the nature and interrelationships of different shocks to different variables. The way to do this was to estimate the variance-covariance matrix of the shocks. In order to do this, the model was simulated dynamically into the future, using the extended path method to include rational expectations into the model.

If the understanding of the nature of the shocks was not achieved, the analysis would become a "black box" with not much practical appeal. The analysis of the shocks would thus provide useful insights into the working of the world economy. However, it should be remembered that correlations between two variables do not necessarily imply causal relations between them. Even Taylor (1993, p. 108) admits that sometimes the most satisfactory explanation to a correlation between two variables is a missing third factor explaining the behavior of more than one variable.

The conclusion of Taylor is that the variance-covariance matrix shows considerable differences in the size of the shocks in the different types of shocks and in different countries. The matrix also shows high degree of correlation between shocks of same variables between different countries.

4.2.3 Simulation results

Taylor is mainly interested in obtaining "elasticity type" information: the percentage changes in output, employment, or other variables that occur in response to a given percentage change in a policy instrument. Since the multi-country model is neither log-linear nor linear, the initial starting values for the variables and for the period over which one conducts policy experiments can in principle make a difference for these percentage changes. Taylor claims that in practice, however, the time period and the level of the variables make only small differences for the model. Moreover, to a close approximation, the percentage changes do not depend on the level of the variables or the time period. Hence, although these simulations focus on a particular ten-year period (1975:1 through 1984:4), they can be interpreted as applying to any other ten-year period, for example, from 1993:1 through 2002:4. (Taylor 1993, p. 136)

Taylor concentrates (naturally) on the United States, and only monetary policy design issues are considered. Taylor skips fiscal policy rules by stating that a similar approach could be used to study them.

The simulations were carried out as stochastic simulations. The shocks were assumed to have normal distribution, where the variance-covariance matrix was estimated from the structural residuals. The simulations were also carried out with the shocks drawn randomly from the residuals themselves.

The simulation results imply, among other things, that flexible exchange rate regime economies are more stable than fixed exchange rate regimes, especially concerning output and price levels. The results also suggest that there is not much need to coordinate or negotiate on the design of monetary policy rules among countries. The results indicate also that, for flexible exchange-rate systems, nominal income rules that weigh output deviations as well as price deviations in the central banks' reaction function frequently perform better than price rules. (Taylor 1993, pp. 250–251)

A general conclusion that can be drawn from these results is that placing some weight on real output in the interest rate reaction function is likely to be better than a pure price rule. A more general rule that places less weight on real output than a nominal-output rule stabilizes the price level better than a nominal-income rule. (Taylor 1993, p. 251)

Although new results have been reported since this relatively old study (1993), it was these simulation exercises, where Taylor developed his famous rule-of-a-thumb, the "Taylor Rule", which is still used by various central banks (Taylor 1993b).

Chapter 5

Model applicability to new countries

A research group based at the School of Economics of Fundação Getúlio Vargas São Paulo (FGVSP) in Brazil is developing a macroeconomic model of Brazilian economy. As an exercise, Taylor's multi-country model was also meant be tested with Brazilian data. Other countries to be added to the framework would have been Argentina, Mexico, United States, and European Union as one economic area.

This work started as a study to test the model applicability for Brazil. However, due to the limited resources, the test could not be finished. This chapter reports the work that was done and what will be the next steps in the future research.

5.1 Literature review and technical study

Taylor's book (1993) is in itself a complete reference and instructions book for his multi-country model. Also, literature on more recent models were studied, but since the focus of the study was Taylor's model, most of the references were found from Taylor himself.

The model was also re-estimated by same econometric program that Taylor used, TSP, in order to verify that the procedures were correct. However, the subroutine to to calculate the rational expectations values during the estimation was not available, and an equivalent routine was not yet written, so the re-estimation of the equations with rational expectations was left to be done in the future. Instead, actual values of the variables were used, and in many cases, the results differed substantially, but the rough scale of the parameters did not change.

Also, the 4,700 lines of Fortran code of the simulation program was studied. The program appeared to be very inflexible, that is, difficult to adjust to different schemes such as adding another country. However, the study of the program was very useful in gaining understanding about the simulation itself. With enough resources (=time), similar program with more flexibility could be programmed.

5.2 Data collection

All the relevant data for Brazil was collected from the free-access database of the Institute of Applied Economic Research (IPEA) of Brazil's Ministry of Planning, Budgeting and Administration, Ipeadata (http://www.ipeadata.gov.br). Ipeadata has up-to-date data of all Brazilian macroeconomic data that is available. All the variables could not be found in their disaggregate form. Thus, total consumption was collected instead of durables, non-durables and services consumtion. Similarly, total fixed investment was collected in place of non-residential equipment and structures and residential investment.

Since Taylor used quarterly data, yearly data was deemed too inaccurate, although it could also be used, providing that there were enough sample years. Moreover, when quarterly data was not available, monthly data was collected. Any econometric program can easily transform monthly data into quarterly data. The only thing for the user to define is if the data is to be averaged or summed.

For the parameter estimation exercise, yearly data for the United States was collected from the World Bank's World Development Indicators online service (http://devdata.worldbank.org/dataonline/), accessible through the library of the Helsinki School of Economics. This data was also transferred into quarterly form, though the variability between the quarters was not achieved. However, the data proved sufficient for the exercise.

5.3 Parameter estimation

The following step was to estimate the equations for the Brazilian data. In the following subsections, the estimation is reported for the different equations. In this preliminary stage, no further actions were made, but the need for it was acknowledged whenever there were irregularities to Taylor's estimations.

Due to the lack of a proper rational expectations estimation routine (it is widely known that rational expectations models are estimated case by case, that is, each model is solved by its author with a tailored software or a subroutine to a more general econometric software), in this exercise the rational expectations values were substituted by the actual values, whenever applicable.

The estimation results are summarized in Table 5.1.

5.3.1 Wage determination

The first equation proved to be the most complicated. Although the model is linear in parameters, it becomes more complicated when the unknown variable LX is solved. It has to be solved into a reduced form, factored into a polynomial product and then estimated with maximum likelihood methods. This is usually tailored to each separate case, and this time there were no enough resources to do the necessary programming, so the wage equations estimation was left as a future exercise.

Also, more recent discussion about the Taylor's wage setting model seem to be in favour of a different formulation. Fuhrer and Moore (1995) and Blanchard (1998) state that while Taylor's wage setting model (Taylor 1980) imply that prices are sticky, it also implies that the inflation rate is so flexible that monetary policy can drive a positive rate of inflation to zero with virtually no loss of output. Fuhrer and Moore show that Taylor's model is not consistent with the dynamic interaction of inflation and output. They propose a contracting specification where agents care about relative real wages.

In a recent study of the Brazilian economy, Bonomo and Brito (2001) use Fuhrer and Moore's (1995) specification to model the Brazilian wage determination, arguing that nominal rigidity is not sufficient in itself to create the inflationary inertia that is observed in economies. Another option would be to relax the rational expectation hypothesis, on which the whole Taylor's multi-country model is based.

5.3.2 Aggregate price adjustment

This estimation was straightforward, with all parameters except the autocorrelation coefficient ρ being significant, and also the R^2 value being relatively high, 0.85. The DW statistic was, however, smaller than in Taylor's estimations, being 0.6, whereas in Taylor's estimations it was always approximately at the value of 2. So autocorrelation may not be corrected with the first-order autoregressive error, but it has to be inspected more carefully.

5.3.3 Import prices

As this equation requires the price levels of the other countries in the model, which were not yet decided, the equation could not be estimated as such. However, some test estimations were made with the original U.S. import price equation, substituting the average foreign price LFP with a price of each country. In half of the cases, the estimation gave essentially similar values, so it was decided to try to estimate Brazil's equation using the U.S. price level as a proxy for the average foreign price. The United States' participation of Brazil's foreign trade has been about 20 percent during the period 1994–2004, for which the data was collected.

The estimation, however, failed, with the R^2 value being 0.07. At least this did not seem to be due to autocorrelation, since the DW statistic was 2.0. Thus, more experiments are needed with the import price equation.

5.3.4 Export prices

This estimation did not go too well, either, but at least the parameters were significant, with the exception of the trend variable and the autocorrelation coefficient. (R^2 was 0.25, substantially lower than in the original work). Interestingly, Taylor does not report in his work the standard errors for the autocorrelation parameters. Like for several countries in the original study, the parameter for the foreign price variable LFP (in this case the U.S. price) was not significant.

5.3.5 Term structure of interest rates

Since the nonlinear two-stage least squares estimation method used again the average foreign price LFP as instrument variable, the original equation for the United States was re-estimated using any foreign price index (Canada, France, Germany, Italy, Japan and the United Kingdom) instead of the average foreign price, and the estimation results were essentially equal. Thus, it was once again deemed sufficient to use the U.S. price index as the instrument variable.

Although the R^2 value was only 0.26 compared for example to the 0.47 of the original U.S. equation, both equation parameters were statistically significant. The low DW value was also present in Taylor's estimations, but he does not comment on that.

5.3.6 Consumption demand

The estimation went relatively well with the R^2 being 0.88. However, only the parameter for permanent income YP was statistically significant. Regarding this equation, like the two following equations, it has to be remembered that the "real" rational expectations values were not used but instead the actual values were used.

5.3.7 Fixed investment

This equation had two significant parameters, those of the lagged variable and the permanent income. On the other hand, the R^2 was lower, 0.56.

5.3.8 Inventory investment

The R^2 value was even lower, 0.28, and the only significant parameter was that of the lagged value.

5.3.9 Exports and imports

The U.S. price level and national product were used instead of the average variables. The estimations had the R^2 values of 0.96 and 0.92 for exports and imports, respectively. All the parameters for exports were significant, with only the lagged variable being significant for imports.

5.3.10 Money demand

The money demand equation was also estimated successfully, with the R^2 value being 0.85, and all parameters except the short-term interest rate RS being significant.

5.3.11 Conclusions

Most of the estimations with the Brazilian data were successful. This is a good basis for the future work, where the remaining equations will be estimated. However, the validity of the wage setting equations needs to be studied and possibly tested with alternative specifications.

5.4 Future work

In the future, the macroeconomic data for the remainder of the countries in the proposed model will be collected. When calculating the aggregate variables, the weights of each country or group need to be defined. Taylor does not mention in his book how he defined the weights, but they seem to conform to the size of the economies.

The subroutine to implement the rational expectations solutions needs to be programmed. However, there exist also more modern econometric software that can handle equations with rational expectations.

After the parameter estimation, the nature of the simulations need to be decided. This will be done by the FGVSP research group.

5.5 Conclusions

Due to the lack of resources, the exercise to test Taylor's multi-country model's applicability to Brazil was not carried out fully. It was left as a future exercise.

The data for Brazil was collected and some estimation exercises were already conducted with encouraging results. The remaining data needs to be collected in order to carry out the test. Another alternative would be estimating the equations for Brazil with the other G-7 countries.

If the parameters (equations) for the G-7 countries could be assumed to be stable, the simulations could be made with the parameters estimated for those countries during a different period. However, Taylor himself admits that the estimations were very sensitive to the time period and some adjustments had to be already done in order to estimate all the parameters.

Moreover, this would in any case mean collecting the updated data for the countries in order to calculate the average variables. Also, the weight to Brazil would have to be defined.

Also, Taylor's simulation program was written in Fortran, and adding another country to the simulation would practically mean rewriting a program with over 4,700 lines of code. The program was written in a way that it is very difficult to modify the program in order to include another country or use the program for another number of countries.

Table 5.1: Estimation results for Brazilian data. The values in parenthesis are the respective t statistic values

Aggregate price adjustment

Cons.	$LP(-1)$	LW	$LPIM(-1)$	T	ρ	SE/R^2	DW
-0.292	0.0552	0.785	0.160	0.0268	-0.0230	0.048	0.6
(-4.51)	(*)	(8.33)	(2.28)	(11.13)	(-0.13)	0.85	

Export prices

Cons.	$LPEX(-1)$	LP	LFP	T	ρ	SE/R^2	DW
0.0597	0.452	0.548	-	-0.0016	0.0629	0.056	2.0
(2.17)	(*)	(3.36)		-1.27	0.17	0.25	

Term structure of interest rates

Cons.	b	SE	R^2	DW
-0.0878	1.225	0.047	0.26	0.11
(-7.06)	(4.54)			

Consumption demand

Cons.	$CD(-1)$	YP	RRL	SE	R^2	DW
4.272	0.101	0.0289	-39.112	9.91	0.88	2.1
(0.16)	(0.35)	(3.59)	(-0.66)			

Fixed investment

Cons.	$IX(-1)$	YP	RRL	SE	R^2	DW
-2.609	-0.544	0.0137	13.083	6.83	0.56	1.4
(-0.11)	(-3.97)	(3.23)	(0.18)			

Inventory investment

Cons.	$II(-1)$	Y	$Y(-1)$	RRL	SE/R^2	DW
-14.051	-0.652	0.252	-0.236	138.440	8.59	1.4
(-0.47)	(-3.02)	(1.04)	(-0.95)	(0.96)	0.28	

Exports

Cons.	$LEX(-1)$	$LPEX - LPIM$	LYW	SE	R^2	DW
-13.352	0.426	0.644	2.132	0.123	0.96	1.7
(-3.31)	(6.51)	(6.42)	(4.26)			

Imports

Cons.	$LIM(-1)$	$LPIM - LP$	LY	SE	R^2	DW
-0.495	1.003	0.248	0.0286	0.136	0.92	1.5
(-0.26)	(7.80)	(1.77)	(0.17)			

Money demand

Cons.	$LMP(-1)$	RS	LY	SE	R^2	DW
3.143	0.368	-0.0986	0.246	0.042	0.85	1.6
(3.88)	(2.71)	(-0.37)	(2.16)			

(*) The parameters were calculated from other parameters in the equations due to the parameter restrictions and thus do not have t-values

Chapter 6

Conclusions

6.1 Results of the study

This study concentrated on understanding the macroeconometric multi-country model used in Taylor (1993) to simulate different financial and fiscal shocks to the economies. An attempt to use the same model for another context (Brazil) was made. Some of the parameters needed for the simulation were estimated, but the estimation was not completed. Also, the initial results indicate a possibility that the model might not be transferable without some modifications.

In the following, the experiences of the study are reflected. Thereafter, some criticism and suggestions for alternative research are presented.

6.2 Reflections

This work has been a useful exercise in understanding the complexity of building a multi-country macroeconometric model and using it in simulations, and in trying to apply an existing model to a new area and data. The first difficulties were met already trying to replicate the original model. In this case, the data were already given so the only difficulty was running the estimations. However, since rational expectations models are not easy to estimate, some difficulties were met already here.

The next step is to complete the parameter estimation and to find suitable data for the country or countries of interest. In Brazil's case, macroeconomic data were given by Ipeadata (http://www.ipeadata.gov.br). However, all is not done when the data are collected. The estimation is not guaranteed to succeed. There are also other things to consider. For example, what other countries should be taken to the model. The simplest solution would be retaining the original countries. The following question is how to make the interdependent equations compatible among themselves. And even the simplest solution would already require reprogramming the whole model again.

At least the author's understanding about macroeconometric multi-country models increased. In the beginning of the project, the subject was something not seen at the economic courses of the Helsinki School of Economics. And after the second read-through, the subject was still not clear. Only after studying several sources about the same subject, a clearer picture could be formed.

The mathematics behind rational expectations are also beyond the normal level taught for undergraduates. Although a conceptual understanding was achieved, much more time and effort would be needed to understand the concept fully. Fortunately for the author, nonlinear models oblige numerical methods, and their implementation is somewhat simpler task, although some algorithm programming skills are still required.

However, this brings forth one interesting question: with all the high-end computer software that does all the task of estimating for the user, could there be something valuable lost in the process? Studying through the extensive Fortran program code and comparing it to the theoretical models or algorithms presented in scientific papers made the author understand more about the whole simulation process. Can the omission of this phase jeopardize the ability to interpret the simulation results and the (almost always appearing) anomalities?

Studying the program code also points out the "expert judgement" that the economist who built the program has used. This prepares the user to already scrutinize the results with that judgement in mind.

6.3 Criticism

The Lucas Critique uses the implications of the theory of rational expectation for the use of evaluating economic policy. According to Lucas Critique set forth in 1976, prediction based on historical data would be invalid if some policy change alters the relationship between relevant variables. If the policy change alters the relationship between the variables, then the historical relationship between variables would differ from the future relationship. (Lucas 1976)

Whitley's argument is that "The response to the Lucas critique was that it was overstated and, in any case, could be accommodated by the use of rational expectations." (Whitley 1994, p. 46) However, even the International Monetary Fund has started to use the new GEM model that is independent of the rational expectations hypothesis, unlike their older Multimod model. Fair's model (2004) assumes, too, that the expectations are not rational. He tests the rational expectations hypothesis, but it is rejected in most cases.

The parameters of the models provide exact instructions for policymakers. However, policymakers are unlikely to start following them mechanically. Even Taylor admits, that they may be justified to do that. (Taylor 1993, p. 280)

Output gap is also a difficult definition. In Taylor's work, output gap was defined as a difference between real GNP and trend (potential) GNP, but the definition of potential GNP is already controversial.

Another issue with the models is that in reality, there are no exogenous variables. Some factors are always behind every variable. And it is these variables that produce shocks to the economies. If this could be understood better, maybe the solutions to the problems economists (and policymakers) pursue, would be nearer. Of course, there are some shocks where humans cannot have any influence, like natural catastrophies (and to some extent, wars).

Have the models become too complex already? If the question is to determine a certain policy's effect on certain economic variables, shouldn't these two (or more) variables be separated and the solution be searched with the simplest ways? Modelling economies is always a generalization, and often not a good one.

"The outstanding question is then whether too much resource input goes into forecasting relative to its benefits." (Whitley 1994, p. 199) Some might even think that economists would be more valuable to economies working on stone quarries.

6.4 Suggestions for alternative research

Since the nonlinear multi-country models cannot be studied analytically, an alternative could be used to simulate the behaviour of the economies with a fuzzy model. Fuzzy modelling does not require anything else than sufficient data so that the system can build a logic where all the interrelationships of different variables are taken into account implicitly. Theoretical aspects of models will not be considered but sometimes they are even limiting the models, since every aspect cannot be taken into account. Thus, fuzzy models may provide better simulation results.

Of course, using fuzzy models means losing the analytical understanding behind the models. But since the models can never replicate all interrelationships and causalities, this may also be an advantage, depending on which is more important in the study, understanding the theory or getting a good replication of the actual economy or economies. Also, the fuzzy system can also be de-constructed and there may appear some unexpected causalities that would not have been observed otherwise.

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Appendix

Model variables

Financial variables

RS	short-term interest rate
RL	long-term interest rate
RRL	real interest rate (see equations for definition)
Ei	exchange rates (U.S. cents per foreign currency of country i)
M	money supply, $M1$ definition

Real GNP (or GDP) spending components

Y	real GNP (or GDP)
C	consumption (total)
CD	durable consumption
CS	services consumption
CN	nondurables consumption
INS	nonresidential structures investment
INE	nonresidential equipment investment
IR	residential investment
II	inventory investment
IF	fixed investment (total)
IN	nonresidential investment (total)
IR	residential investment (total)
EX	exports in income-expenditure identity
IM	imports in income-expenditure identity
G	government purchases of goods and services

Variables relating to GNP

YP	permanent income (see equations for definition)
YW	weighted foreign output
YT	trend or potential output
T	time trend
YG	percentage gap between real GNP and trend GNP

Wages and prices

W	average wage rate
X	"contract" wage rate (constructed from average wage index, see equations)
P	GNP (or GDP) deflator
PIM	import-price deflator
PEX	export-price deflator
PW	trade-weighted foreign price (foreign currency units)
EW	trade-weighted exchange rate (foreign currency/domestic currency)
FP	trade-weighted foreign price (domestic currency units)

NB: A capital "L" in front of a variable means a natural logarithm of the variable.

Model equations

$$
\begin{aligned}
LX_i &= \pi_{i0}LW_i + \pi_{i1}LW_i(+1) + \pi_{i2}LW_i(+2) + \pi_{i3}LW_i(+3) \\
&\quad +\alpha_i[\pi_{i0}YG_i + \pi_{i1}YG_i(+1) + \pi_{i2}YG_i(+2) + \pi_{i3}YG_i(+3)], \\
\text{where} &\quad LW_i = \pi_{i0}LX_i + \pi_{i1}LX_i(-1) + \pi_{i2}LX_i(-2) + \pi_{i3}LX_i(-3) \\
&\quad YG = \beta_1 YG(-1) + \beta_2 YG(-2) + \beta_3 \\
LP_i &= h_{i0} + h_{i1}LP(-1) + h_{i2}LW_i + h_{i3}LPIM_i(-1) + h_{i5}T + U_{pi} \\
U_{pi} &= \rho_{pi}U_{pi}(-1) + V_{pi} \\
\text{with} &\quad h_{i1} + hi2 + h_{i3} = 1 \\
LPIM_i &= k_{i0} + k_{i1}LPIM(-1) + k_{i2}LFP_i + U_{mi} \\
U_{mi} &= \rho_{mi}U_{mi}(-1) + V_{mi} \\
\text{with} &\quad k_{i1} + ki2 = 1 \\
LPEX_i &= \beta_{i0} + \beta_{i1}LPIM(-1) + \beta_{i2}LP_i + \beta_{i3}LFP_i + \beta_{i4}T + U_{xi} \\
U_{xi} &= \rho_{xi}U_{xi}(-1) + V_{xi} \\
\text{with} &\quad \beta_{i1} + \beta i2 + \beta_{i3} = 1 \\
LE_i &= LE_i(+1) + 0.25 * (RS_i - RS_0) + U_{ei} \\
U_{ei} &= \rho_e U_{ei}(-1) + V_{ei} \\
RL_i &= b_{i0} + \frac{1-b_i}{1-b_i^9}\sum_{s=0}^{8} b_i^s RS_i(+s) \\
CX_i &= c_{i0} + c_{i1}CX_i(-1) + c_{i2}YP_i + c_{i3}RRL_i, \\
\text{where} &\quad YP_i = \sum_{s=0}^{8}(0.9)^s Y_i(+s) \\
&\quad RRL_i = (RL_i - LP_i(4) - LP_i)\exp(gT) \\
IX_i &= d_{i0} + d_{i1}IX_i(-1) + d_{i2}YP_i + d_{i3}RRL_i \\
II_i &= e_{i0} + e_{i1}II_i(-1) + e_{i2}Y_i + e_{i3}Y_i(-1) + e_{i4}RRL_i \\
LEX_i &= f_{i0} + f_{i1}LEX(-1) + f_{i2}(LPEX_i - LPIM_i) + f_{i3}LYW_i \\
LIM_i &= g_{i0} + g_{i1}LIM(-1) + g_{i2}(LPIM_i - LP_i) + g_{i3}LY_i \\
L\left(\frac{M_i}{P_i}\right) &= a_{i0} + a_{i1}L\left(\frac{M_i(-1)}{P_i(-1)}\right) + a_{i2}RS_i + a_{i3}LY_i
\end{aligned}
$$